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A General Formula for Noncohesive Suspended Sediment Transport

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ABSTRACT

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A simple and robust suspended load transport formula for noncohesive sediment is presented for application to river, estuarine, and coastal environments with the use of depth-averaged models. The formula is based on an exponential profile for the concentration and assumes a constant velocity over depth to simplify the calculations. These assumptions were validated with a large data set, including data with a steady current, wave and current interaction, and breaking waves. The formula has two parameters: the mean sediment diffusivity over depth and the bottom reference concentration. The sediment diffusivity is estimated assuming a linear combination of mixing because of breaking waves and the energy dissipation in the bottom boundary layer from the mean current, waves, or both. The bottom reference concentration is a function of the Shields parameter. Overall, the formula developed in this study yields the best agreement with the compiled data set compared with a number of existing formulas for estimating the suspended load.

ADDITIONAL INDEX WORDS: *Suspended load, concentration, profile, reference concentration, diffusion parameter, current, waves, breaking waves, noncohesive sediment.*

INTRODUCTION

Accurate prediction of noncohesive sediment transport rates is an important element in morphological studies for river, estuarine, and coastal environments. Depth-averaged (2DH) models are widely employed nowadays and generally allow for the use of one sediment transport formula only in the entire domain. Such a formula needs to be robust and reliable in a wide range of conditions. In estuarine and coastal environments, the process of sediment transport becomes increasingly complex because of the presence of oscillatory flows and the interaction between steady and oscillatory flows. For example, for longshore sediment transport, the influence of short waves is expressed as sediment stirring, which increases bed shear stress and the vertical mixing coefficient (diffusion) for sediment in suspension (Bijker, 1968; Van Rijn, 1993; Watanabe, 1982). The development of practical sediment transport models still has a strong empirical character and relies heavily on physical insights in combination with quantitative data obtained in laboratory and field studies.

The earliest formulas were mainly based on the concept that the sediment transport rate for steady uniform flows can be related to bottom shear stress (Einstein, 1950; Engelund and Hansen, 1972; Meyer-Peter and Müller, 1948) and assumed that bed load transport prevailed. However, when the bottom shear stress is large enough, the sediment particles can be lifted, put in suspension, and transported in large quantities by the current. Thus, suspended load is often dominant for fine sediment (median grain size $d_{50} < 0.5$ mm) under medium shear stress, under the presence of bed forms, or with wave stirring. The depth-averaged volumetric suspended load transport q_{ss} is herein defined (see Nielsen, 1992, p. 201) as the integrated product of the velocity u and the concentration c from the edge of the bed load layer ($z = z_R$) to the water surface ($z = h$), averaged in time, yielding,

$$q_{ss} = \overline{\int_{z_R}^{h} c(z,\ t)u(z,\ t)\ dz} \tag{1}$$

where h is the water depth, z is a vertical coordinate, and $\bar{x}$ is the time-averaged value of the variable x.

Assuming that variables u and c can be decomposed into two components, *i.e.,* a time-averaged component $\bar{u}$ and $\bar{c}$ and an oscillating component $\tilde{u}$ and $\tilde{c}$, $u(z,\ t) = \bar{u}(z) + \tilde{u}(z,\ t)$ and $c(z,\ t) = \bar{c}(z) + \tilde{c}(z,\ t)$, then Equation (1) becomes:

$$q_{ss} = \int_{z_R}^{h} \bar{c}(z)\bar{u}(z)\ dz + \overline{\int_{z_R}^{h} \tilde{c}(z,\ t)\tilde{u}(z,\ t)\ dz} \tag{2}$$

where the first term corresponds to the current-related suspended load and the second term to the wave-related sus-

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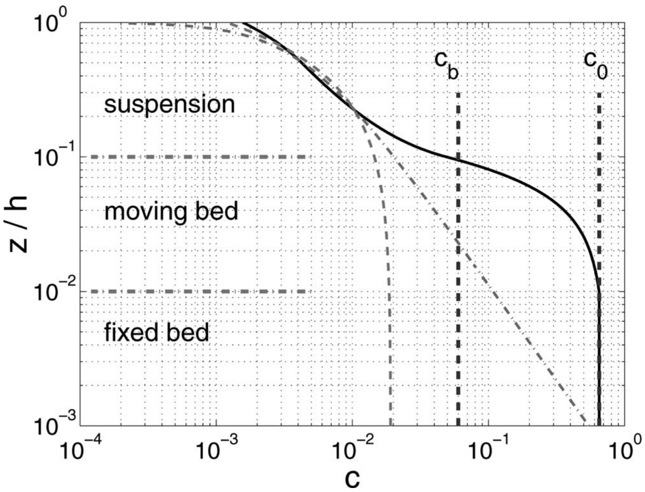

Figure 1. Typical concentration profile (full line) from the fixed bed to the water surface on the basis of experimental results by Dohmen-Janssen and Hanes (2002), together with the exponential (dashed line) and power law (dash-dotted line) profiles fitted to the suspended concentration.

pended load. Steady conditions are generally assumed to simplify the problem, so time-averaged values $\bar{u}(z)$ and $\bar{c}(z)$ are used. Therefore, an accurate estimate of the total suspended load requires correct predictions of the mean current velocity and the concentration profile. Even if the unsteady part of the suspended sediment transport is significant for some cases (phase lag effects over ripples; see Van der Werf and Ribberink, 2004), in this study, we will focus on the steady part because most of the experimental data measured this quantity only.

The overall objective of this study was to develop a reliable and general formula for predicting suspended load transport under a wide range of river, estuarine, and coastal conditions. Such a formula is proposed for 2DH models in which only an average value of the velocity over the water depth is provided. This paper is organized as follows: The proposed formula is first developed, then the experimental data used for this study are described, together with a validation of the hypothesis underlying the proposed formula. The study and calibration of the two main parameters, which are the reference concentration c_R and mean diffusivity ϵ, are presented. Finally, conclusions concerning the derived net suspended load transport formula and its predictive capability are discussed.

DEVELOPMENT OF A SUSPENDED LOAD FORMULA

On the basis of experimental results by Dohmen-Janssen and Hanes (2002), Figure 1 presents a typical concentration profile in which sheet flow and suspended load coexist. They observed a drop in concentration at the top of the sheet flow layer and defined the edge of the sheet flow (bed load) layer in which the concentration $c = c_b = 0.08$ (the maximum volume concentration is defined as $c_0 = 0.65$). It appears that close to the bed load, both power law and an exponential profile underestimate the concentration. On the other hand, these two theoretical profiles yield very similar results for the suspended concentration. Indeed, compared with the experimental data compiled (see *Experimental Data* section), both exponential or power law profiles could be fitted to the experimental data with a small relative error.

Because the use of a power law profile for the sediment concentration requires a reference level ($z = z_R$), which induces an additional parameter and thus even more uncertainty (because z_R is often arbitrarily chosen), an exponential law profile was preferred ($z_R = 0$ could then be assumed), assuming a constant value for the sediment diffusivity over depth ϵ. By solving the mass conservation equation for the steady equilibrium of a particle under gravity and hydrodynamic forcing, the following profile for the sediment concentration is obtained,

$$c(z) = c_R \exp\left(-\frac{W_s}{\epsilon} z\right) \tag{3}$$

where c_R is the bottom reference concentration, W_s is the settling velocity, and the parameter W_s/ϵ determines the suspension conditions. In determining q_{ss}, following the simplified approach by Madsen, Tajima, and Ebersole (2003), the vertical variation in u can be neglected (also discussed further in the *Validation of the Hypothesis* section). The suspended sediment load is thus found to be

$$q_{ss} = U_c c_R \frac{\epsilon}{W_s}\left[1 - \exp\left(-\frac{W_s h}{\epsilon}\right)\right] = U_c F(c_R, \epsilon) \tag{4}$$

where $U_c = \bar{u}$ is the depth-averaged velocity and the function F determines the quantity of sediment available. In solving the integral, the ratio $W_s h/\epsilon$ is often assumed to be large, implying that the exponential term $\exp(-W_s h/\epsilon) \approx 0$ or $F \approx c_R\epsilon/W_s$.

In the case of wave and current interaction, Equation (4) can be modified to take into account possible sediment transport in the direction of the waves (see Figure 2),

$$\begin{aligned} q_{ssw} &= (U_{cw,\text{onshore}} - U_{cw,\text{offshore}})F(c_R, \epsilon) \\ q_{ssn} &= U_c \sin\varphi F(c_R, \epsilon) \end{aligned} \tag{5}$$

where the subscript "w" indicates the direction of the wave and the subscript "n" indicates the perpendicular direction; $U_{cw,j}$ is the root mean square value of the velocity $[u(t) = u_w(t) + U_c \cos\varphi]$ over the half period $T_{w,j}$, for which the subscript j should be replaced by either *onshore* (for $u(t) \geq 0$) or *offshore* (for $u(t) < 0$); and φ is the angle between the wave and current direction (see Figure 2). For a steady current alone, Equation (5) reduces to Equation (4), and for sinusoidal waves alone, Equation (5) yields zero transport.

Thus, the two main parameters to estimate for calculation of the suspended load are the bottom reference concentration and mean sediment diffusivity over the water depth. Here, various data sets were used for model development with steady and oscillatory flows, including wave breaking. The aim of this paper was to develop and validate a formula that

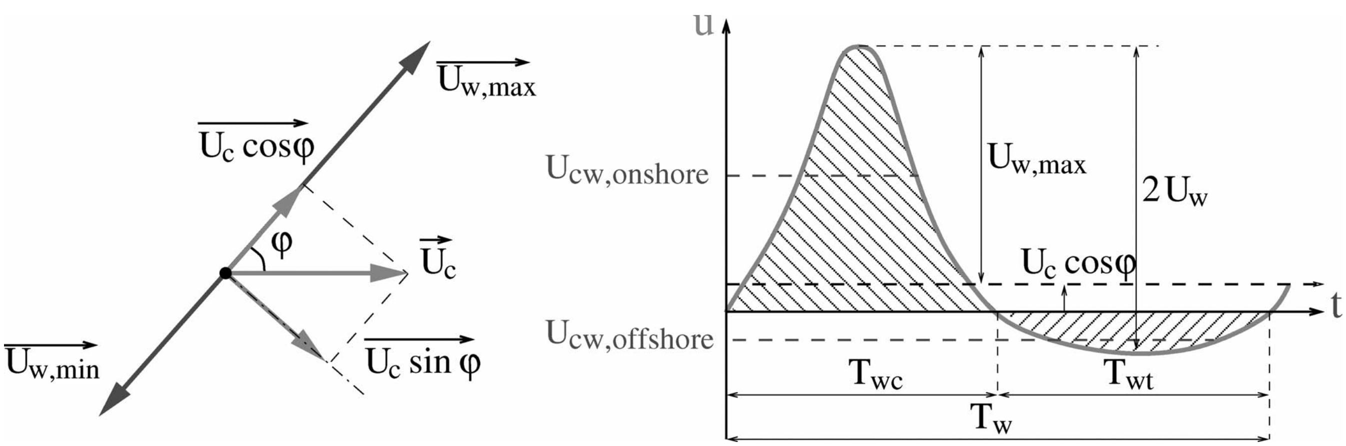

Figure 2. Definition sketch for the depth-averaged velocities.

gives accurate results when both waves and current are present in the nearshore zone. Thus, the effects of a steady current, waves, and breaking waves on sediment diffusivity and the reference concentration were investigated carefully to include all the main hydrodynamic parameters and provide robust predictive formulas.

EXPERIMENT DATA

Selection of the Data

To investigate mean diffusivity over depth, bed reference concentration, and resulting suspended sediment transport in steady conditions, as well as for waves and current combined, a wide range of existing data sets were compiled and analyzed. Depending on the experiments, velocities were measured with impeller flow meters, pitot tubes, acoustic Doppler probes (ADP), laser Doppler velocity meters, or electromagnetic current meters. Time-averaged concentrations were measured with suction (pump) samplers and, more recently, with optical probes (optical backscatter sensors), conductivity probes, and ADP. It is obvious that the different instruments and the precision of the instruments affect the quality and uncertainties of the results. It is difficult, however, and not the purpose here, to discuss in detail the uncertainties induced by the measurements in the different experiments. As pointed out previously, the limit between the bed load and the suspended load is difficult to establish. For most of the cases, concentrations were not measured close enough to the bed, or the instruments were not able to measure very high concentrations. The suspended load was estimated by the reference level proposed by VAN RIJN (1993), *i.e.,* $z_R = \max(H_r/2, k_{sg})$, where H_r is the ripple height and $k_{sg} \approx 2d_{50}$ is the grain-related roughness height. An approximate power law regression was used for the experimental data when information was not available close to the bed.

For a steady current, Table 1 summarizes the data sets and lists the type of flow motion and sediment properties. Similarly, for waves and current combined, Table 2 summarizes the data sets, presenting the type of experiment, wave and current conditions, and sediment properties.

For all the experiments presented in Table 2, sand with a relative density $s = \rho_s/\rho = 2.65$ was used (ρ_s and ρ are the sediment and water densities, respectively). Most of these data sets were obtained from the data compilation provided by the SEDMOC European Union research project (VAN RIJN *et al.,* 2001). For wave–current interaction, only the cases in which the mean current (preferably the entire velocity profile) was estimated could be used to calculate the total suspended load. The number of measurement points over the water depth are also shown in Table 2, for both the sediment concentration and the time-averaged velocity, to indicate the spatial resolution of the data. In the cross-shore direction, the

Table 1. *Data summary for experiments on suspended sediment transport under steady flow.*

Authors	Location	Flow Type	No. of Profiles	d_{50} (mm)	b (m)	Fr	u_{*c} (m/s)
Anderson (1942)	Enoree River, USA (1940–41)	River data	23	0.7	15	0.15–0.25	0.02–0.07
Barton and Lin (1955)	Fort Collins, CO, USA	Tilting flume	26	0.18	1.2	0.2–0.9	0.02–0.08
Laursen (1958)	Iowa, USA (1961–63)	Tilting flume	12	0.4, 1	0.9	0.25–0.60	0.02–0.09
Scott & Stephens (1966)	Mississippi River, USA (1961–63)	River	23	0.4	500	0.11–0.16	0.05–0.13
Culberton, Scott, and Bennet (1972)	Rio Grande River, USA (1965–66)	River	22	0.18–0.33	20	0.2–0.6	0.05–0.15
Voogt, Van Rijn, and Van den Berg (1991)	Krammer beach, The Nederlands (April 1987)	Tidal channel	60	0.22–0.35	300	0.1–0.5	0.03–0.15
Peet (1999; see Van Rijn *et al.*, 2001)	Wallingford, UK	Duct experiments	24	0.08–0.20	0.6	0.2–0.4	0.01–0.14

d_{50} = median grain size; b = width of the river/flume; Fr = Froude number; u_{*c} = current-related shear velocity.

Table 2. *Data summary for experiments on suspended sediment transport under oscillatory flow.**

Authors	Location	Flow Type	No. of Profiles†	No. of Measurements‡	d_{50} (mm)	h (m)	U_c (m/s)	U_w (m/s)§	T_w (s)§	H_r (m)	L_r (m)
Bosman (1982), Steetzel (1985)	DHL, The Netherlands	Wave flume	70 (50, 16)	6–15/3–4	0.10	0.1–0.65	0.10–0.32	0.13–0.30	1.4–2.0	0.01–0.03	0.08
Nielsen (1984)	Australian beaches (1980–82), Australia	Field	65 (39, 43)	5–7/—	0.11–0.62	0.8–1.8	0–0.54	0.28–0.80	5.3–14.4	0–0.20**	0–1.5**
Steetzel (1984), Van der Velden (1986)	DHL, The Netherlands	Small water tunnel	259 (259, 0)	5–11/—	0.10–0.36	0.4	0	0.07–0.65	1.0–7.0	0.005–0.1	0.011–0.55
Dette and Uliczka (1986)	Hannover, Germany	Large wave flume	11 (0, 0)	8–10/—	0.33	0.9–2.6	0	0.95–1.65	6.0	—	—
Kroon (1991)	Egmoud beach (1989–90), The Netherlands	Field	31 (11, 31)	5–8/3–5	0.30–0.47	0.4–1.5	−0.55–0.97	0.20–0.91	3.1–12.6	0.005–0.05	0.15–0.75
Havinga (1992)	VinjeBasin, Delft, The Netherlands	Basin	28 (28, 28)	7–10/10	0.10	0.40–0.43	0.10–0.32	0–0.80	2.1–2.3	—	—
Ribberink and Al Salem (1994)	DHL, Delft, The Netherlands	Large water tunnel	71 (71, 0)	5–12/—	0.21	0.8	0	0.2–1.5	2.0–12.0	0–0.35**	0–3.0**
Chung Grasmeijer, and Van Rijn (2000)	Deltaflume, DHL, Delft, The Netherlands	Large wave flume	19 (19, 14)	5–8/5	0.16–0.33	3.5–4.5	−0.04–−0.02	0.56–0.67	6.6–7.1	0.03–0.05	0.25–0.75
Voulgaris and Collins (2000)	Bournemouth beach, Caswell Bay, Rhossili Bay, UK	Field	12 (12, 0)	—/—	0.21–0.33	0.4–2.1	0.01–0.10	0.16–0.40	3.2–9.1	—	—
SEDMOC data set (Van Rijn *et al.*, 2001) - Vessem-	Estern Scheldt estuary (1983–84), The Netherlands	Field	70 (70, 70)	6–10/1	0.15	0.7–4.0	0.05–0.65	0.02–0.40	2.0–3.2	0.05	—
SEDMOC data set (Van Rijn *et al.*, 2001)	Grote Speurwerk (45m), DUT, Delft, The Netherlands	Wave flume	62 (62, 19)	6–10/2–3	0.15–0.29	0.49–0.55	0.16–0.35	0.14–0.60	2.4–2.8	—	—
SEDMOC data set (Van Rijn *et al.*, 2001)	Grote Speurwerk (35m), DUT, Delft, The Netherlands	Wave flume	125 (81, 58)	4–10/9–12	0.10–0.22	0.29–0.60	0.07–0.45	0.17–0.55	1.2–2.7	0.002–0.029	0.006–0.20
SEDMOC data set (Van Rijn *et al.*, 2001)	Deltaflume, DHL, Delft, The Netherlands	Large wave flume	57 (30, 0)	3–13/—	0.19–0.24	0.7–3.4	−0.18–0	0.67–1.46	2.6–5.0	0–0.04**	0–1.0**
Bayram *et al.* (2001)	Sandy-Duck (1996–98), South Carolina, USA	Field	66 (25, 66)	6–9/3	0.18–0.20	1.2–8.6	0.04–1.32	0.71–2.13	8.0–12.8	—	—
Wang, Eversole, and Smith (2002)	LSTF, Vicksburg, Mississippi, USA	Large basin	14 (0, 14)	5–14/6–9	0.22	0.10–0.40	0–0.18	0.27–0.45	1.5, 3.0	—	—

* — = Not measured or not available, DHL = Delft Hydraulic Laboratory, DUT = Delft University of Technology, LSTF = Large Scale Sediment Transport Facility.
† Values in the parentheses are no. of nonbreaking cases and no. of cases in which the suspended load can be estimated.
‡ Number of concentration measurements of the water depth/number of time-averaged velocity measurements over the water depth.
§ For random waves, the orbital wave velocity U_w is computed from the root mean square wave height and wave period $T_w = T_p$.
** Flat bed (bed form height H_r and length L_r are assumed equal to zero.

Table 3. *Prediction (Pred.) of suspended load transport from Equation (4) and the fitted valeu (exponential profile) to the observed data for* c_R *and* ϵ $[f(q_{ss}) = \log |q_{s,pred}/q_{ss,meas}|]$.

Conditions	No.	Pred. (%) ×2	Pred. (%) ×5	$f(q_{ss})$ Mean	$f(q_{ss})$ SD
Steady current	187	99	100	0.03	0.09
Waves and current	322	66	91	−0.16	0.38
Breaking waves	151	72	93	−0.18	0.32

velocity was averaged from the bottom to the trough of the waves. Thus, it corresponds to the mean undertow.

An important question in nearshore sediment transport is to know whether bed load or suspended load prevails. For the lower regime and upper regime (sheet flow with nonbreaking waves), bed load has been observed to prevail over suspended load (Dohmen-Janssen and Hanes, 2002). For medium regimes, when the bottom is covered by bed forms and when waves are breaking, it is often assumed that suspended load prevails (Nielsen, 1992, pp. 201–206). With the Camenen and Larson (2005) formula for bed load (not measured for all the collected data), this assumption was confirmed. For 80% of the collected data, the suspended load largely prevailed over bed load ($q_{ss}/q_{sb} > 10$), especially for very fine sediments and when waves were present. Bed load appeared to prevail mainly with coarse sediment (when the dimensionless grain size $d_* = [(s-1)g/\nu]^{1/3}d_{50} > 15$, in which g is the acceleration of the gravity and ν is the kinematic viscosity of water).

Validation of the Hypothesis

To validate the two main hypotheses underlying the proposed formula, a comparison was made between the observed suspended load and Equation (4) with the fitted values to the observed data on c_R and ϵ for each experimental case (least squares method used to fit the exponential profile). For the data with current only (see Table 1), very good agreement was observed for all data sets. In Table 3, the main results of the comparison are provided—*i.e.,* the percentage of data correctly predicted within a factor of 2 or 5 (Pred. ×2 and Pred. ×5, respectively) and the mean value and standard deviation of the function $f(q_{ss}) = \log|q_{ss,pred}/q_{ss,meas}|$, where $q_{ss,pred}$ and $q_{ss,meas}$ are the predicted and measured suspended load values, respectively. The table shows that 99% of the data are well predicted within a factor of 2. This means that the assumptions of an exponential concentration profile and a constant velocity over depth are sufficiently accurate to estimate the suspended load in the steady current case.

With waves and current combined, the results are not as good as for the steady current data. Only 66% and 91% of the data (72% and 93% in the case of breaking waves) are predicted within a factor of 2 and 5, respectively (see Table 3). Indeed, the suspended load is quite often underestimated. Large uncertainties occurred when estimating the suspended load, both for the actual data and the fitted data. Knowing the concentration profile and use of the mean current velocity instead of the velocity profile does not significantly affect the suspended load. It induces an overestimation of less than 5% for a logarithmic profile (longshore current) and an underestimation of 10% to an overestimation of 20% in more complex flows (undertow). The most important assumption for the sediment transport seems thus to be the exponential concentration profile. For some cases, it appears that a power law profile fits better to the data, especially close to the bed. For many cases—*e.g.,* the Grote Speurwerk (45m) data set—no measurements of the bed features are available. The estimation of the measured suspended load is also strongly linked to the chosen reference level; if bed forms are present, z_R is considered to be equal to half of the measured or estimated bed form height (Van Rijn, 1984b). When the ripple characteristics were not measured, the Soulsby and Whitehouse (2005) formulas were used. This might explain the underestimation observed for some data sets.

SEDIMENT DIFFUSIVITY

The sediment diffusivity ϵ is a fundamental parameter for estimating the concentration profile. It is a function of bottom roughness and associated shear stress, agitation (mainly from waves), and settling velocity.

A General Equation for the Sediment Diffusivity

In a study on infilling of navigation channels, Kraus and Larson (2001) employed an exponential concentration profile to estimate the suspended load transport and assumed, following Battjes (1983), that vertical mixing is proportional to the energy dissipation of wave breaking. Then, to employ a general formula for sediment diffusivity, it seems natural to assume that

$$\epsilon = \left(\frac{D}{\rho}\right)^{1/3} h \tag{6}$$

where D is the total effective dissipation and

$$D = k_b^3 D_b + k_c^3 D_c + k_w^3 D_w \tag{7}$$

in which the energy dissipation because of wave breaking (D_b) and bottom friction from current (D_c) and waves (D_w) are simply added and k_b, k_c, and k_w are coefficients. The coefficient k_b mainly corresponds to an efficiency, whereas k_c and k_w are related to the Schmidt number (ratio between the vertical eddy diffusivity of the particle ϵ and the vertical eddy viscosity ν_v). Typically, $D_b > D_w > D_c$, and in many cases, only the largest dissipation needs to be considered. Still, the formula for ϵ is simplified because mixing should vary through the water column.

The energy dissipation in the bottom boundary layer because of a current can be written

$$D_c = \tau_c u_{*c} \tag{8}$$

where τ_c and u_{*c} are bottom shear stress and shear velocity from the current only, respectively. This expression deviates from the standard way of defining dissipation by a current, which should be expressed as the product between the shear stress τ_c and the mean velocity U_c. However, the use of u_{*c} instead of U_c yields the same result as the classical mixing length approach (Equation [9]).

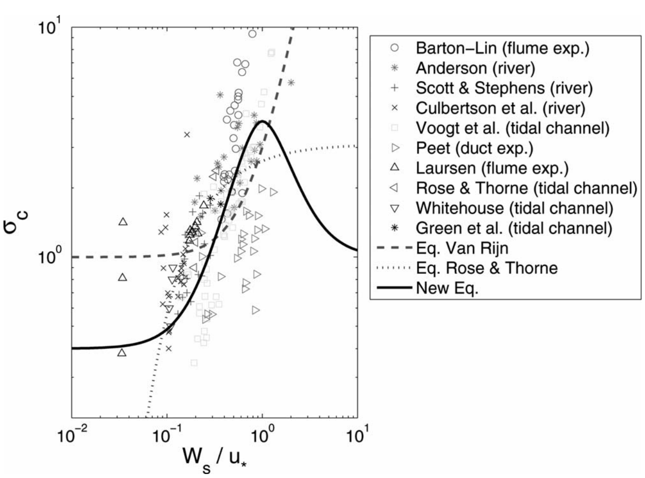

Figure 3. Estimation of the Schmidt number σ_c as a function of the ratio W_s/u_{*c}. **For a color version of this figure, see page 682.**

$$\epsilon_c = k_c\left(\frac{D_c}{\rho}\right)^{1/3} h = k_c u_{*,c} h \tag{9}$$

Similarly, the instantaneous energy dissipation in the bottom boundary layer because of wave motion $D_{wt}(t)$ can be expressed as the product between the instantaneous bed shear stress $\tau_{wt}(t)$ and the instantaneous shear velocity $u_{*wt}(t)$. The sediment diffusivity as the result of waves can thus be written as an average over the wave period:

$$\epsilon_w = k_w\left(\frac{D_w}{\rho}\right)^{1/3} h = k_w u_{*,w} h \tag{10}$$

where D_w $(= \tau_w u_{*w})$, τ_w, and u_{*w} are the wave-related energy dissipation, maximum bottom shear stress, and maximum shear velocity, respectively.

Effects of a Steady Current

For suspended sediment in a steady current, ROUSE (1938) observed that a parabolic equation for the vertical sediment diffusivity best fits the experimental data. Assuming that the parabolic profile is a correct approximation for vertical sediment diffusivity, the relationship in Equation (11) is obtained,

$$k_c = \frac{\sigma_c}{6}\kappa \tag{11}$$

where σ_c is the Schmidt number (also defined as a β-factor by NIELSEN and TEAKLE [2004] or VAN RIJN [1984b]). In general, σ_c is supposed to be a constant and $\sigma_c = 1$. However, SUMER and DEIGAARD (1981) and VAN RIJN (1984b) pointed out that centrifugal forces in fluid eddies cause sediment grains to be thrown outside of the fluid eddies, which increases σ_c. Another reason presented by FREDSØE and DEIGAARD (1992, pp. 231–234) for why σ_c might deviate from unity is sediment settling out of the surrounding water before the water loses its earlier composition by mixing. ROSE and THORNE (2001) added that the estimation of σ_c might be affected by the settling velocity, which varies because of turbulence (a smaller particle settles faster in turbulence; see MURRAY, 1970; NIELSEN, 1993). Finally, NIELSEN and TEAKLE (2004) proposed a Fickian diffusivity model showing the effect of the size of the particle on the Schmidt number. They also argued that the Schmidt number might be less than unity for fine particles over a flat bed because the mixing length could be smaller for these particles than for the fluid (MUSTE and PATEL, 1997). To maintain a consistent approach between the analysis of different data sets, the von Karman constant was assumed to equal the clear water value = 0.4, even though some evidence suggests that this parameter is reduced by the presence of suspended sediment (on the basis of the assumption that the Schmidt number equals 1). These variations are thus included in the Schmidt number.

Table 4. *Prediction (Pred.) of the Schmidt number for the case of a steady current [*$f(\sigma_c) = \log |\sigma_{c.pred}/\sigma_{c.meas}|$*].*

Authors	Pred. (%) ×2	Pred. (%) ×5	$f(\sigma_c)$ Mean	$f(\sigma_c)$ SD
Van Rijn (1984b)	78	100	−0.02	0.25
Rose and Thorne (2001)	75	98	−0.02	0.32
Equation (12)	76	99	−0.04	0.26

On the basis of measurements by COLEMAN (1981), VAN RIJN (1984b) suggested an expression for σ_c that is a function of the ratio between the settling velocity and the current-related shear velocity. With the exponential law to derive the sediment diffusivity, it is also possible to estimate the coefficient σ_c from the experimental concentration profiles. The experimental results are presented in Figure 3 together with the Van Rijn and Rose and Thorne formulas and the new equation proposed in this paper (Equation [12]), as well as additional data from ROSE and THORNE (2001) and VAN RIJN (1984b).

To put forward a relationship that gives physically meaningful results for all cases, it should be considered that the Van Rijn equation is correct only for $W_s/u_{*c} < 1$. For $W_s/u_{*c} \gg 1$, the Schmidt number must be equal to unity because suspension is negligible (sediments do not affect the flow). Thus, a new expression for σ_c is proposed in Equation (12) (see also Figure 3),

$$\sigma_c = \begin{cases} A_{c1} + A_{c2}\sin^2\left(\dfrac{\pi}{2}\dfrac{W_s}{u_{*c}}\right) & \text{if } \dfrac{W_s}{u_{*c}} \le 1 \\ 1 + (A_{c1} + A_{c2} - 1)\sin^2\left(\dfrac{\pi}{2}\dfrac{u_{*c}}{W_s}\right) & \text{if } \dfrac{W_s}{u_{*c}} > 1 \end{cases} \tag{12}$$

where $A_{c1} = 0.40$ and $A_{c2} = 3.5$.

Overall, the new equation yields good predictions compared with the data (see Table 4). Nevertheless, data from the PEET (1999, see Van Rijn *et al.*, 2001) experiments are, in general, overestimated, perhaps because measurements were only carried out close to the bed ($z < h/5$).

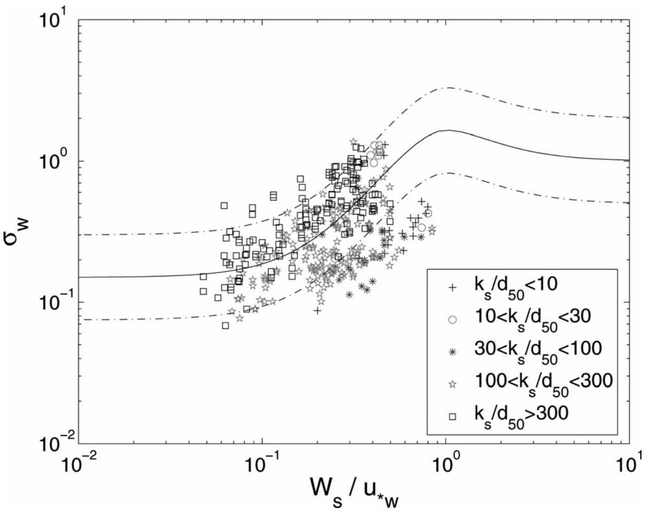

Figure 4. Estimation of the Schmidt number σ_w with Equation (14) as a function of the ratio W_s/u_{*w}, with the roughness ratio k_s/d_{50} emphasized. **For a color version of this figure, see page 682.**

Effects of Nonbreaking Waves

Following the results obtained for the steady current, the relationship in Equation (13) for k_w is obtained (the factor $2/\pi$ results from time averaging assuming a sinusoidal wave),

$$k_w = \frac{\sigma_w}{3\pi}\kappa \tag{13}$$

where σ_w is the wave-related Schmidt number.

With the data sets presented in Table 2, the total shear velocity was estimated assuming that the shear velocities due to the current and waves can be added linearly. For most of the cases (wave flumes), the shear stress from the current is much smaller than the shear stress from the waves and, thus, can be neglected. The total roughness height k_s was estimated by the method of Soulsby (1997), adding the grain-related, form drag, and sediment transport components $k_{sg} = 2d_{50}$, k_{sf}, and k_{ss}, respectively. The roughness height k_{sf} was obtained with the Kim (2004) formula, which is a function of the bed form characteristics, whereas k_{ss} was obtained with the Wilson (1966, 1989) formula.

Following the method used for sediment diffusivity because of a current (see previous section), the correction factor (Schmidt number) was assumed to be a function of the ratio W_s/u_{*w}. The general tendency of the data appears similar to the steady current case (see Figures 3 and 4),

$$\sigma_w = \begin{cases} A_{w1} + A_{w2}\sin^2\left(\frac{\pi}{2}\frac{W_s}{u_{*w}}\right) & \text{if } \frac{W_s}{u_{*w}} \le 1 \\ 1 + (A_{w1} + A_{w2} - 1)\sin^2\left(\frac{\pi}{2}\frac{u_{*w}}{W_s}\right) & \text{if } \frac{W_s}{u_{*w}} > 1 \end{cases} \tag{14}$$

where $A_{w1} = 0.15$ and $A_{w2} = 1.5$.

Table 5. *Prediction (Pred.) of the diffusivity for waves only [f(ϵ_w) = log|$\epsilon_{w,pred}/\epsilon_{w,meas}$|].*

Authors	Pred. (%) ×2	Pred. (%) ×5	$f(\epsilon_w)$ Mean	$f(\epsilon_w)$ SD
Dally and Dean (1984)	14	51	0.67	0.32
Van Rijn (1993)	38	76	0.29	0.52
Nielsen (1992)	56	90	−0.31	0.57
Equations (10) and (14)	69	100	0.09	0.28

The wave-induced Schmidt number (see Equation [14]) is often much smaller than that found for a steady current. However, because the friction velocity from waves is generally much larger than that from a current, the mixing attributable to waves is also much larger. In Figure 4, there seems to be a relationship between σ_w and the roughness ratio k_s/d_{50}. Because this roughness ratio (and the total shear stress) is calculated from empirical formulas, and not estimated directly from the experimental data (contrary to the data with current only), the relationship between ϵ_w and W_s/u_{*w} exhibits larger scatter.

The results obtained by Equations (10) and (14) are better than those for the other studied formulas (see Table 5). Thus, 69% of the data is correctly predicted within a factor of 2 and 100% within a factor of 5. The percentages of values obtained within a factor of 2 or 5, as well as the mean value and the standard deviation of the function $f(\epsilon_w) = \log|\epsilon_{w,pred}/\epsilon_{w,meas}|$, are presented in Table 5.

It appears that of the existing formula, the one proposed by Dally and Dean (1984), who assumed that the Rouse (1938) expression could be used, presents the least scatter, even if the formula in general overestimates sediment diffusivity. This expression might be correct, but the Schmidt number appears to be much smaller than 1. The more complex formulas introduced by Nielsen (1992, pp. 215–217) or Van Rijn (1993) yield better results (especially for the Nielsen formula), but also larger scatter. The Nielsen formula tends, however, to largely underestimate the results when $U_w/W_s > 18$.

Wave–Current Interaction

Simply adding the sediment diffusivity from the current and waves (Equations [9], [10], [12], and [14]) leads to overestimation compared with the data. This overestimation could be because the Schmidt number should be the same for the current as for the waves. A more physical description of the wave and current interaction should be based on a unique Schmidt number, calculated as an empirical weighted value between σ_c and σ_w,

$$\sigma_{cw} = X_t\sigma_c + (1 - X_t)\sigma_w \tag{15}$$

where $X_t = \theta_c/(\theta_c + \theta_w)$, in which $\theta_c = \tau_c/[(s - 1)gd_{50}]$, and $\theta_w = \tau_w/[(s - 1)gd_{50}]$, which are the current-related and wave-related Shields parameters, respectively.

Table 6 shows the results obtained with the studied formulas. Because larger values are observed in cases with wave and current interaction, the formulas in which the sediment diffusivity was overestimated for waves only provide better

Table 6. *Prediction (Pred.) of the sediment diffusivity in the wave and current interaction case [$f(\epsilon_{cw}) = \log |\epsilon_{cw,pred}/\epsilon_{cw,meas}|$].*

Authors	Pred. (%)		$f(\epsilon_{cw})$	
	×2	×5	Mean	SD
Dally and Dean (1984)	47	88	0.26	0.39
Van Rijn (1993)	38	83	0.19	0.50
Nielsen (1992)	00	01	−1.68	0.80
Equation (9) + Equation (10)	50	85	0.31	0.36
idem with $\sigma_c = \sigma_w = \sigma_{cw}$ [Equation (15)]	65	92	0.08	0.39

Table 7. *Prediction (Pred.) of the reference concentration assuming an exponential sediment concentration profile for the studied data set with steady current only [$f(c_R) = \log(c_{R,pred}/c_{R,meas})$].*

Authors	Pred. (%)		$f(c_R)$	
	×2	×5	Mean	SD
Madsen, Tajima, and Ebersole (2003)	27	50	0.75	0.83
Nielsen (1986, 1992)	13	50	0.64	0.43
Equations (16) and (17)	38	83	−0.12	0.55
Equation (16) with $A_{cr} = 5 \times 10^{-4}$	28	65	0.05	0.77

results (see formulas by Dally and Dean, 1984; Van Rijn, 1993). The Nielsen (1992) formula yields poor results because $U_w/W_s > 18$ for most of the cases. The proposed formula to calculate ϵ_{cw} by Equation (15) yields the best results, although it often overestimates when large bed roughness is computed.

Effects of Breaking Waves

In the case of breaking waves, the energy dissipation was calculated with the energy dissipation of a bore analogy (Svendsen, 1984) with the coefficient $A_\epsilon = 2 \tanh(5\xi_\infty)$ proposed by Stive (1984) to take into account breaker-type effects ($\xi_\infty = m/\sqrt{H_\infty/L_\infty}$ is the Irribaren parameter defined for deep water, m is the mean slope of the beach, and H_∞ and L_∞ are the deep-water wave height and length, respectively). In the random waves case, the coefficient $\alpha_b = \exp[-(\gamma_b h/H_{rms})^2]$ should be added to take into account the percentage of breaking waves (see Larson, 1995), where γ_b is the breaker depth index and H_{rms} is the root mean square wave height, neglecting breaking. As a first approximation, the efficiency coefficient was found to be constant: $k_b = 0.010$. Even if some dispersion exists for the compiled data, Equation (6) yields results as good as for the nonbreaking cases: 72% (96%) of the data are well predicted within a factor of 2 (5), and the mean and standard deviation of the function $f(\epsilon_b) = \log|\epsilon_{b,pred}/\epsilon_{b,meas}|$ is 0.03 and 0.32, respectively.

REFERENCE CONCENTRATION

Einstein (1950) proposed that the reference concentration could be a function of the bed load transport. Following Madsen, Tajima, and Ebersole (2003), the reference volumetric bed concentration can be estimated from the volumetric bed load, assuming $q_s = c_R U_s$, where U_s is the velocity of the bed load layer. The bed load can be written following the results of Camenen and Larson (2005), namely $q_s \propto \theta^{3/2} \exp(-4.5\theta/\theta_{cr})$, where θ is the Shields parameter (the subscript "cr" indicates its critical value for the inception of sediment motion; see Camenen and Larson, 2005; Soulsby, 1997). Madsen, Tajima, and Ebersole (2003) proposed, as a first approximation, that the speed of the bed load layer is proportional to the shear velocity, $U_s \propto \theta^{1/2}$. The bed reference concentration could thus be written as Equation (16),

$$c_R = A_{cR}\theta_T \exp\left(-4.5\frac{\theta_M}{\theta_{cr}}\right) \qquad (16)$$

where θ_T is the transport-dependent Shields parameter and θ_M is the maximum Shields parameter. For current only, $\theta_M = \theta_T = \theta_c$.

Effect of a Current

For the data with a steady current, the coefficient A_{cR} was found to vary from 5×10^{-6} to 4×10^{-2}. The use of a constant mean value $A_{cR} = 5 \times 10^{-4}$ produces results already better than the Madsen, Tajima, and Ebersole (2003) formula (see Table 7). Van Rijn (1984a, 1984b) observed that the reference concentration c_R is a function of the dimensionless grain size d_*, but to a varying power depending on the presence or absence of bed forms. For the compiled data (see Table 1), the dimensionless grain size d_* varies from 1 to 18. Improved results were obtained by calibrating A_{cR} as a function of the dimensionless grain size, as in Equation (17).

$$A_{cR} = 1.5 \times 10^{-3} \exp(-0.2d_*) \qquad (17)$$

The percentages for values obtained within a factor of 2 or 5, as well as the mean value and the standard deviation of the ratio $f(c_R) = \log(c_{R,pred}/c_{R,meas})$, are presented in Table 7. It appears that the Madsen, Tajima, and Ebersole (2003) formula presents correct results compared with the experimental values of c_R assuming an exponential profile. However, this formula seems not to be sensitive enough to the Shields parameter: it gives more or less a constant value for each data set. The Nielsen (1986, 1992, pp. 201–233) formula, although fitted with data on waves only, presents correct results for the laboratory experiments (data from Barton and Lin, 1955; Laursen, 1958; Peet, 1999 [see Van Rijn *et al.*, 2001]). It tends, however, to overestimate the results for the field experiments, even if bed form influence on the Shields parameter were not taken into account because information was not available.

In Figure 5, the predicted reference concentration c_R from Equations (16) and (17) is plotted against the estimated reference concentration assuming an exponential profile. Even if the results are in agreement overall, it seems that sensitivity to the Shields parameter should be larger. The reference concentration is generally overestimated for low shear stresses. The prediction of the reference concentration is significantly improved compared with the Madsen, Tajima, and Ebersole (2003) formula: nearly 40% of the data are predicted within a factor of 2 and 85% within a factor of 5. The mean value of $f(c_R)$ is closer to zero, and its standard

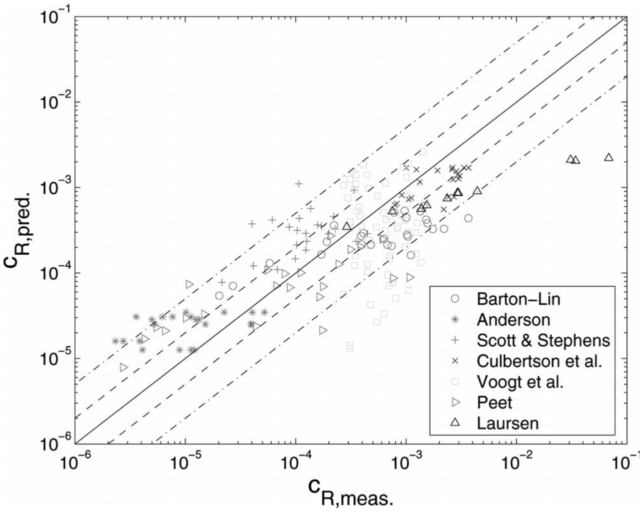

Figure 5. Predicted reference concentration c_R with Equations (16) and (17) *vs.* the estimated reference concentration assuming an exponential profile. **For a color version of this figure, see page 682.**

Table 8. *Prediction (Pred.) of the reference concentration with the waves-only data set [$|U_c| < 0.05$ m/s, $f(C_R) = \log(c_{R,pred}/c_{R,meas})$].*

Authors	Pres. (%) ×2	Pres. (%) ×5	$f(c_R)$ Mean	$f(c_R)$ SD
Nielsen (1986, 1992)	26	60	0.34	1.04
Madsen, Tajima, and Ebersole (2003)	20	45	0.76	0.57
Equations (16) and (17)	31	66	−0.27	0.63
Equation (16) with $A_{cr} = 5 \times 10^{-4}$	31	67	−0.26	0.61

deviation is reduced compared with the previous formulas. However, some dispersion appears for the data sets of PEET (1999, see Van Rijn *et al.*, 2001) and VOOGT *et al.* (1991), which produces the negative value for the mean of $f(c_R)$.

Effect of Nonbreaking Waves

For the waves only case, according to the results of CAMENEN and LARSON (2005) for bed load transport, the mean shear stress $\theta_{w,m}$ is used for the transport-dependent term $\theta_T = \theta_{w,m}$, whereas the maximum wave shear stress θ_w is used for the critical shear stress ($\theta_M = \theta_w$). The mean Shields parameter attributable to the waves is defined as $\theta_{w,m} = ½f_w/[(s-1)gd_{50}][\int_0^{T_w} u_w(t)^2\,dt]$, whereas the maximum Shields parameter is defined as $\theta_w = ½f_wU_w^2/[(s-1)gd_{50}]$, where f_w is the wave-related friction coefficient and $\theta_{m,w} = 0.5\theta_w$ in the case of a sinusoidal wave.

For the data with waves prevailing ($|U_c| < 0.05$ m/s; see Table 2), a comparison was made between the different formulas studied and the data (see Table 8 and Figure 6). Equation (16) with Equation (17) for current only still presents correct results, although the dispersion is larger. The effect of grain size seems not to be as significant as for the results with current only. Similar results are obtained with Equation (16) in which $A_{cR} = 5 \times 10^{-4}$. However, the range of values on d_* was larger for the data set with current only (d_* from 1.0 to 18). In the case of current only, $d_* < 5$ for 40% of the data, whereas 95% of the data was below this value for waves only. This result could explain the difference in the results for the current, and especially the difference observed with the use of Equation (16) and $A_{cR} = 5 \times 10^{-4}$ (overestimation for the current data set and underestimation for the wave data set).

Another reason for the large scatter comes from the uncertainties in the bed form characteristics (measured or estimated) and in the total shear stress calculation. A_{cR} is observed to be a function of the ripple height, H_r, or, more specifically, of the roughness height ratio k_s/d_{50} (see Figure 6). Finally, the MADSEN, TAJIMA, and EBERSOLE (2003) formula (as well as Equation [16] with $A_{cR} = 5 \times 10^{-4}$) again shows reasonable results because it is not as sensitive to d_* and θ. In Figure 6, the roughness height ratio is emphasized and shows that the calculation of the total shear stress induces large uncertainties (assuming that the reference concentration c_R should not be a function of the ripple height or the roughness ratio, but only of the total shear stress). It appears that the more overestimated/underestimated the reference concentration is, the higher/smaller the roughness ratio.

Wave–Current Interaction

For the wave–current interaction, the intuitive Shields parameters to be used in Equation (16) are $\theta_T = \theta_{cw,m}$ and $\theta_M = \theta_{cw}$. To simplify the calculations, the mean and maximum Shields parameters from the wave–current interaction can be obtained simply through addition: *i.e.,* $\theta_{cw,m} = (\theta_c^2 + \theta_{w,m}^2 + 2\theta_{w,m}\theta_c \cos\varphi)^{1/2}$ and $\theta_{cw} = (\theta_c^2 + \theta_w^2 + 2\theta_w\theta_c\cos\varphi)^{1/2}$, respectively, where φ is the angle between the wave and current directions.

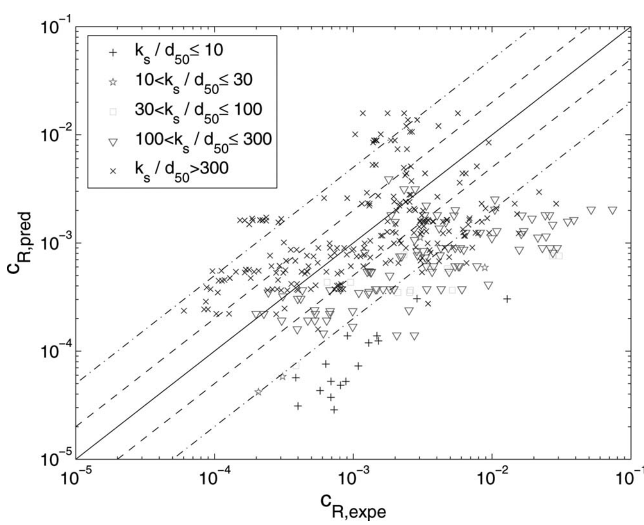

Figure 6. Reference concentration c_R estimated from the data compiled (with waves only, $|U_c| < 0.05$ m/s) *vs.* c_R calculated with Equations (16) and (17) (roughness ratio emphasized). **For a color version of this figure, see page 682.**

Table 9. *Prediction (Pred.) of the reference concentration from the studied data set with wave–current interaction [parentheses hold the results for breaking waves only.* $f(c_R) = log(c_{R,pred}/c_{R,meas})$*].*

Authors	Pred. (%) ×2	Pred. (%) ×5	$f(c_R)$ Mean	$f(c_R)$ SD
Nielsen (1986, 1992)	42 (30)	68 (60)	−0.25 (0.30)	0.73 (0.90)
Madsen, Tajima, and Ebersole (2003)	02 (08)	10 (37)	1.11 (0.90)	0.43 (0.50)
Equations (16) and (17)	52 (62)	82 (90)	0.17 (0.01)	0.53 (0.43)
Equations (16) with $A_{cr} = 5 \times 10^{-4}$	57 (59)	85 (88)	0.07 (−0.04)	0.50 (0.45)

Table 10. *Prediction (Pred.) of suspended load transport in the steady current case [*$f(q_{ss}) = log(q_{ss,pred}/q_{ss,meas})$*].*

Authors	Pred. (%) ×2	Pred. (%) ×5	$f(q_{ss})$ Mean	$f(q_{ss})$ SD
Bijker (1968)	24	45	0.60	1.04
Engelund and Hansen (1972)	31	55	0.65	0.85
Bailard (1981)	33	72	0.32	0.69
Van Rijn (1984b)	30	69	−0.27	0.98
This work with equation (16)	37	79	−0.10	0.57

The percentages for values obtained within a factor of 2 or 5, as well as the mean value and the standard deviation of the ratio $f(c_R) = \log(c_{R,pred}/c_{R,exp})$, for the wave–current interaction are presented in Table 9 for the investigated formulas and Equation (16). The Nielsen formula presents better results compared with the waves only case, but it is still very scattered (because it is a function of the ripple characteristics). On the other hand, the MADSEN, TAJIMA, and EBERSOLE (2003) formula does not have so much scatter, but it generally overestimates, as does the present formula with $\theta_T = \theta_{cw,m}$. However, the new formula still presents the best results among those studied. It appears that the computation of the mean wave and current Shields parameter $\theta_{cw,m}$ significantly influences the results. Compared with the waves only cases, the formula yields better results but generally overestimates. The use of Equation (16) with $A_{cR} = 5 \times 10^{-4}$ produces surprisingly similar estimates.

Effect of Breaking Waves

As a first approach, it can be assumed that wave breaking does not affect the reference concentration, but only the sediment diffusivity. Indeed, as shown by NIELSEN (1992, p. 219), the turbulence induced by the breakers generally occurs in the upper part of the water column; it should not influence the bottom concentration significantly. The reference concentration could, however, be enhanced by the breakers in the case of plunging waves, in which the generated turbulent jet might penetrate to the bottom.

The data sets presented in Table 2 involve many experimental cases in which breaking waves occurred. Table 9 presents the prediction results depending on the chosen formula. Even if the results are scattered, Equations (16) and (17) present the best results among the formulas studied, with 62% of the data correctly predicted within a factor of 2 and 90% within a factor of 5. No clear effect from the type of breaker could be observed from the collected data.

SUSPENDED LOAD

The use of Equation (4) together with the expressions for sediment diffusivity (Equation [6]) and the reference concentration (Equation [16]) allows for prediction of the suspended load for a steady current, waves and current combined, and breaking waves.

Comparison with Data for Current Only

A comparison between the predicted and observed suspended sediment load is presented in Table 10 and Figure 7. In general, the proposed formula (Equation [4]) shows correct behavior. The obtained results, however, appear to be highly dependent on the estimate of the reference concentration when a large dispersion is observed. Indeed, the equation proposed for c_R for a steady current only (*i.e.,* Equations [1] and [17]) produces much better results than the use of a constant value for A_{cR} ($= 5 \times 10^{-4}$). An increase in accuracy of nearly 10% and a decrease in the standard deviation by 10% can be observed.

In Figure 7, similar behavior as that of the reference concentration predictions is observed: a general overestimation for the ANDERSON (1942), PEET (1999, see Van Rijn *et al.*, 2001), and SCOTT and STEPHENS (1966) data sets and a general underestimation for the BARTON and LIN (1955) and LAURSEN (1958) data. A comparison with other semiempirical formulas found in the literature showed that the proposed relationship significantly improves the results when Equation (16) is used for the reference concentration.

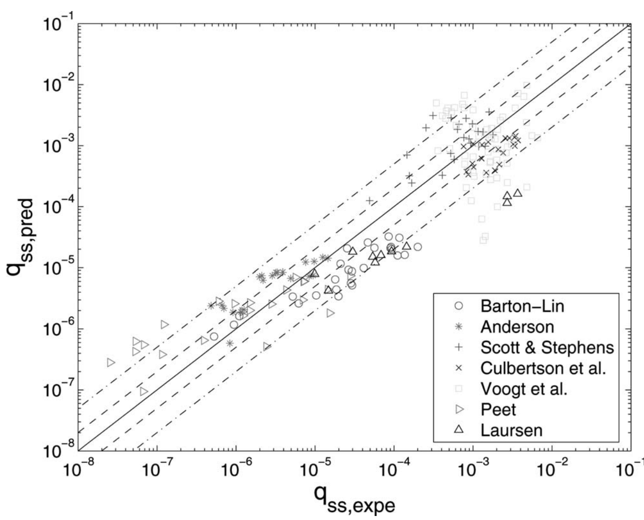

Figure 7. Comparison between the observed suspended sediment load and the calculated suspended sediment load with Equation (4) and the predicted values for c_R (Equation [16]) and ϵ (Equation [7]) for the current only case. **For a color version of this figure, see page 682.**

Table 11. *Prediction (Pred.) of suspended load transport in the interaction between current and waves [parentheses show the results for breaking waves only,* $f(q_{ss}) = log(q_{ss.pred} / q_{ss.meas})$*].*

Authors	Pred. (%)		$f(q_{ss})$	
	×2	×5	Mean	SD
Bijker (1968)	10 (23)	40 (59)	0.83 (0.43)	0.60 (0.70)
Bailard (1981)	19 (30)	58 (74)	0.65 (0.47)	0.52 (0.51)
Van Rijn (1993)	30 (23)	70 (62)	−0.22 (−0.03)	0.74 (0.69)
Equations (4), (6), and (16)	44 (42)	77 (76)	0.23 (−0.05)	0.61 (0.60)

Comparison with Data for Waves and Current Combined

A comparison between the predicted and observed suspended sediment load for the wave–current interaction is presented in Table 11 and Figure 8. It appears that the proposed formula (Equation [4]) presents overall good results. Again, the results are highly dependent on the estimation of the reference concentration and, thus, as shown in the *Reference Concentration* section, on the estimation of the roughness height and total shear stress. Moreover, plunging breaking waves can induce larger values on the reference concentration, as discussed in the *Effect of Breaking Waves* section. Because the prediction of sediment diffusivity is generally less scattered, if an overestimation/underestimation is observed for the prediction of the reference concentration (Grote Speurwerk (35m), Vessem, BAYRAM *et al.* [2001]/KROON [1991] data sets), the same observation can be made for the resulting suspended load.

The percentages for values obtained within a factor of 2 and 5, as well as the mean value and the standard deviation of the ratio $f(q_s) = \log(q_{s,pred}/q_{s,exp})$, are presented in Table 11 for nonbreaking and breaking waves (results in parentheses are for breaking waves). For the wave–current interaction without breaking waves, the VAN RIJN (1993) formula yield the best, though underestimated, results. The large number of parameters and their relative complexity might explain the

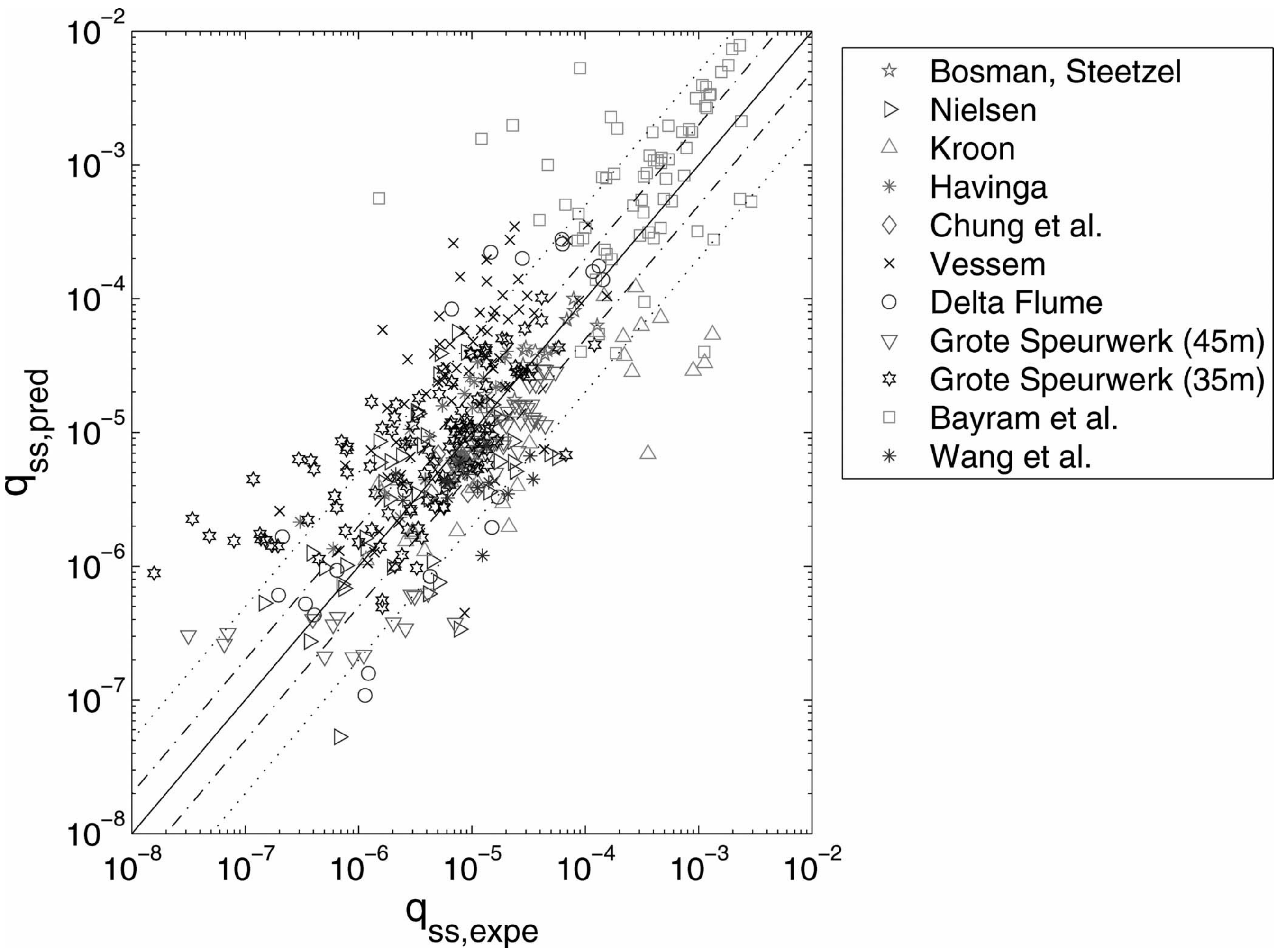

Figure 8. Comparison between observed and predicted values of the suspended sediment load in the wave–current interaction case according to Equation (4). **For a color version of this figure, see page 682.**

observed scatter because it is more sensitive to any given parameter. The result obtained with the Van Rijn (1993) formula is poorer when waves are breaking. On the other hand, it appears that the Bailard (1981) formula presents the best results among the studied formulas for breaking waves. This formula was calibrated to estimate the suspended load in the surf zone, thus yielding good predictions under breaking waves. The Bailard (1982) formula also produces less dispersion. Indeed, this formula is not sensitive to shear stress (only an average friction coefficient is introduced), and it is simple enough to reduce the dispersion of the results. However, no improvement of the results could be obtained because the formula is basically only a function of the current and wave velocities at the bottom. Calculation of the friction coefficient also appeared to affect the results of the Bailard (1982) formula. In the same way as the Bijker (1968) formula, the Bailard (1982) formula generally tends to overestimate the prediction.

Equation (4) yields the best results for both nonbreaking and breaking cases. The result appears, however, to overestimate for the case of nonbreaking waves and to underestimate slightly for the case of breaking waves (see the Kroon [1991] data set in Figure 8). Improvements in the results could be obtained with better predictions of the total shear stress and the reference concentration, as shown in the *Effects of Nonbreaking Waves* section.

CONCLUSION

In this paper, we presented a semi-empirical formula to estimate the suspended load resulting from a mean current and waves. Comparison between formula predictions and measured suspended load from a large data set demonstrated that the formula overall produces satisfactory predictions of the transport rate for a wide range of hydrodynamic and sediment conditions. Assuming an exponential profile for the sediment concentration and a constant value over depth for the time-averaged current velocity, the resulting sediment transport rate can be estimated from a simple equation (see Equation [4]). The two main parameters are mean sediment diffusivity over depth and the bottom reference concentration.

A relationship for sediment diffusivity is proposed assuming a linear combination of mixing by breaking waves and energy dissipation in the bottom boundary layer from a mean current, waves, or both (see Equations [6] and [7]). In the boundary layer, dissipation from the current/waves was expressed as the product between a force (bottom shear stress) and a velocity (shear velocity) so that it is consistent with the classical mixing length approach. Formulas to predict the Schmidt number σ were developed for a mean current and waves separately. For mixing by breaking waves, an efficiency coefficient was introduced, and its value was determined through calibration with experimental data.

Following the Madsen, Tajima, and Ebersole (2003) approach, the reference concentration was found to be proportional to the mean Shields parameter, including the effect of the critical Shields parameter introduced by Camenen and Larson (2005). The results displayed considerable scatter, mainly because of uncertainties in the prediction of the total Shields parameter, especially when bed forms were present. Also, as Van Rijn (1993) suggested, the dimensionless grain size d_* was taken into account in the calculation of the reference concentration. It significantly improves the results for cases with current only or with waves only.

The resulting formula for the suspended load is robust and effective, and it gives the best results among the formulas studied in most cases, although some dispersion still exists. Furthermore, because it is a physically based formula, an improvement of the process knowledge (*e.g.,* for the estimation of the total shear stress) could easily be taken into account in the formula and thus improve the results. When the wave-related suspended load prevails (phase lag effects over a rippled bed), Equation (5) can be modified to take into account these effects by adding a coefficient $\alpha_{pl,s}$ that will decrease the characteristic onshore velocity and increase the characteristic offshore velocity.

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